5 Illustrative Examples

In this section, a process for producing 6061-T6 aluminum sheeting at Boeing Aircraft Company will be simulated as an example to illustrate the proposed bootstrap control charts for monitoring the breaking strength of aluminum material. The method of observing strength of 6061-T6 aluminum was described in detail by Birnbaum and Saunders[2,4] and Onar and Padgett[24]. Three data sets from the experiment were reported and studied by Birnbaum and Saunders[2,4]. They fitted the proposed model (the Birnbaum-Saunders fatigue life distribution) to each of the three data sets separately and achieved good fits. The reported data sets were not for the purpose of constructing quality control charts originally and the data were rearranged to sort the data in an increasing order. Thus, the reported data sets can not be used directly. In this example, The first data set of size 101 was transformed to units of hundreds of thousands of cycles to failure and was used to estimate the parameters of the Birnbaum-Saunders model for strength of material as the true population parameters input for in-control process. Twenty subgroups of size 5 strength measurements were simulated independently from this in-control process with the Birnbaum-Saunders model with $\alpha = 0.2795$ and $\beta = 1.358$ (and thus, the first percentile $W_{0.01} = 0.7165839$). These 20 in-control subgroups are reported in Table XIV.

Table XIV: First 20 subgroups-in control.

subgroup	Breaking stress of 6061-T6 aluminum						Breaking stress of 6061-T6 aluminum				
1	1.1400	1.4300	1.0760	2.1130	1.4890	11	1.5180	1.1450	1.4940	0.9917	2.0220
2	1.0800	1.5560	1.6690	1.5950	1.2470	12	2.3460	1.2260	1.0150	1.5920	1.3080
3	2.0660	1.5140	1.1420	0.7382	1.8570	13	2.6250	1.3430	1.6460	1.3690	1.1040
4	1.3410	1.3520	1.7670	1.7080	1.6030	14	1.4320	0.8243	2.0400	1.4170	2.4700
5	1.7540	1.6890	1.3870	0.7842	1.6150	15	1.5510	1.1140	1.6100	1.0470	0.9583
6	1.3370	1.3000	0.9028	1.1880	1.5260	16	1.4730	1.2000	1.3580	1.3870	1.1520
7	1.9810	1.3200	1.5130	1.3380	0.9263	17	1.1590	1.3080	1.8850	0.8899	1.6030
8	1.2090	1.2160	1.3360	1.8450	1.6800	18	1.4900	1.8260	1.2470	1.5060	1.4630
9	1.2970	1.2650	1.6500	1.5860	1.1210	19	1.1670	1.9000	1.8760	1.6510	2.1080
10	1.1150	1.5040	1.6830	1.3160	1.7360	20	1.5870	0.9522	1.1570	0.9660	1.1900

Shape parameter $\alpha = 0.2795$ and scale parameter $\beta = 1.358$ ($W_{0.01} = 0.7165839$)

5.1 MLE bootstrap control charts

The MLE bootstrap control chart for Birnbaum-Saunders percentile with FAR=0.0027 was established by utilizing these 20 subgroups in Table XIV from in-control process following the Steps 2 to 7 in Section 3 with $B = 10,000$ and the LCL and UCL were calculated. And the center line (CL), $\hat{W}_p$, was calculated by using the same 20 subgroup samples in Table XIV from in control process.

Table XV: Second 20 subgroups-after process shift.

subgroup	Breaking stress of 6061-T6 aluminum					subgroup	Breaking stress of 6061-T6 aluminum				
21	0.2802	8.5320	0.6270	0.7666	0.4136	31	4.4740	0.4357	2.3460	0.4436	0.5680
22	0.5193	1.2900	1.6890	2.0510	1.5920	32	0.5459	1.1980	2.1900	3.8360	2.0280
23	2.3440	0.7174	1.3840	0.8069	4.2170	33	0.4348	1.1260	5.9990	1.1450	1.1550
24	0.4155	3.1470	0.5654	1.0830	0.7527	34	1.7790	0.7027	1.2210	3.0870	0.5239
25	1.5110	1.6880	0.3693	2.5640	0.5355	35	1.9460	0.5319	2.6140	1.3990	5.2110
26	0.2187	0.7716	1.1730	0.8068	1.6490	36	2.2010	0.6235	4.5500	0.8861	1.3870
27	1.4360	3.9910	0.9569	1.3440	1.9070	37	0.5653	3.5950	1.4090	0.3026	0.7666
28	1.7260	0.7707	5.8540	1.6310	3.0470	38	1.5310	1.3910	1.6390	3.1030	2.9400
29	0.9880	0.8541	7.2170	3.6320	1.2210	39	0.5322	0.7735	0.8168	0.5365	5.0100
30	4.5620	1.1860	4.4480	1.5750	0.5950	40	1.4360	1.1170	1.0440	1.0380	0.8156

Shape parameter $\alpha = 0.8782$ and scale parameter $\beta = 1.358$ ($W_{0.01} = 0.2260552$).

Assume that after the process had shifted to an out-of-control process that had different shape parameter $\alpha = 0.8782$ (and thus, the first percentile, $W_{0.01} = 0.2260552$, for the out-of-control process), further subgroups of size 5 from the out-of-control process were simulated for monitoring purpose and were reported in Table XV. Figure 1 shows the LCL (0.4538979) and UCL (1.473584) of the MLE bootstrap control chart for the first percentile of Birnbaum-Saunders model under in-control process, and another 20 first percentile statistics from the out-of-control process. The center line (CL) is also superimposed in the figure. The MLE bootstrap control chart with the CL indicates that the sampling distribution for the first percentile of Birnbaum-Saunders distribution is not symmetric. It can be seen clearly that the process has immediately signaled out of control and only four points among 20 points are within the limits and one point above the CL.

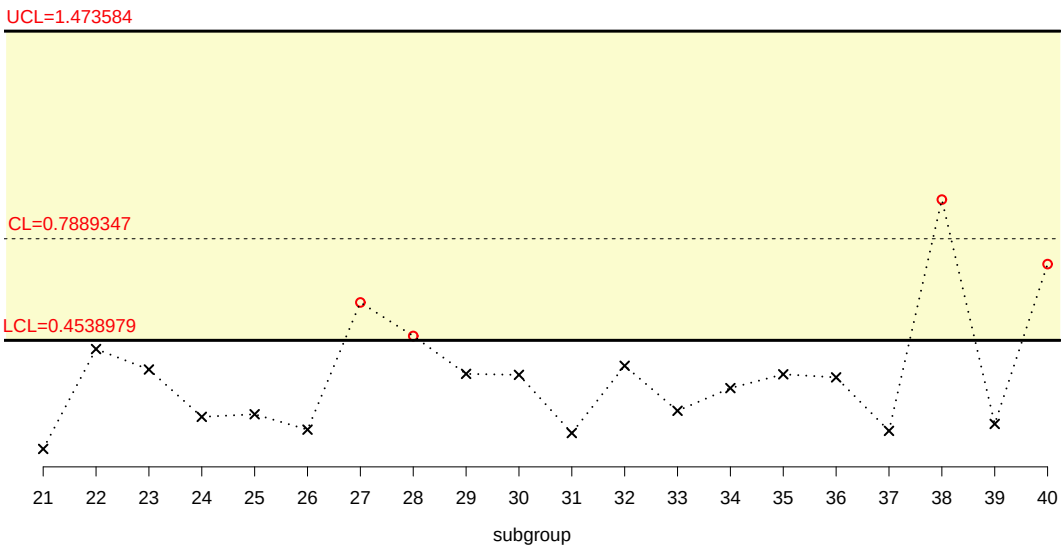

Figure 1: MLE bootstrap Control chart based on 20 subgroups with FAR $\gamma_0 = 0.0027$

5.2 MME bootstrap control charts

Similarly, the MME bootstrap control chart for Birnbaum-Saunders percentile with FAR=0.0027 can be established through the same process but using MME method instead of MLE method. Figure 2 shows the LCL (0.453785) and UCL (1.473601) of the MME bootstrap control chart for the first percentile of Birnbaum-Saunders model. The center line, $\tilde{W}_p$, was calculated by MME method with all 20 subgroup samples in Table XIV from in control process. It can be seen that the MME bootstrap control chart is almost the same as the MLE bootstrap control chart.

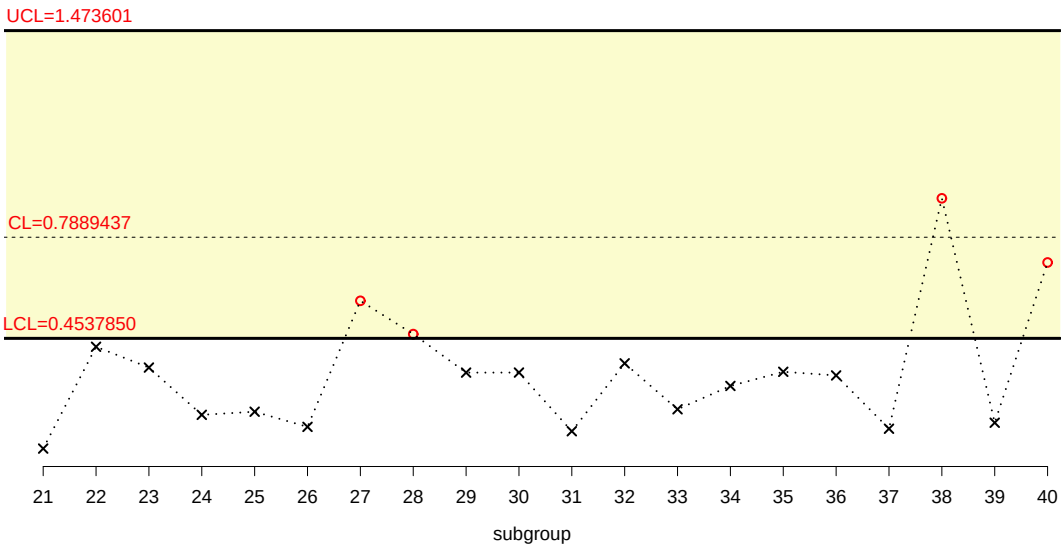

Figure 2: MME bootstrap Control chart based on 20 subgroups with FAR $\gamma_0 = 0.0027$